**Программа утверждена на заседании кафедры теории вероятностей**

**Протокол № 6 от 18 января 2015 г.**

**Рабочая программа дисциплины (модуля)**

1. Код и наименование дисциплины (модуля): МОДЕЛИ СТРАХОВЫХ РИСКОВ (Insurance Risk Models).

2. Уровень высшего образования – специалитет.

3. Направление подготовки: 01.05.01 Фундаментальные математика и механика. Специализация:Фундаментальная математика.

4. Место дисциплины (модуля) в структуре ООП: вариативная часть ООП. Является специальной дисциплиной (спецкурсом) для студентов 3-6 годов обучения, специализирующихся в данной научной области или смежной научной области, спецкурсом по выбору студента.

Освоение дисциплины необходимо для последующего изучения дисциплин образовательной программы: курсовая работа, научно-исследовательская практика, преддипломная практика, выпускная квалификационная работа.

5. Планируемые результаты обучения по дисциплине (модулю), соотнесенные с планируемыми результатами освоения образовательной программы (компетенциями выпускников)

6. Объем дисциплины (модуля) в зачетных единицах с указанием количества академических или астрономических часов, выделенных на контактную работу обучающихся с преподавателем (по видам учебных занятий) и на самостоятельную работу обучающихся:

Объем дисциплины (модуля) составляет 3 зачетных единицы, всего 108 часа, из которых 44 (46\*) часа составляет контактная работа студента с преподавателем (34 (36\*) часа занятия лекционного типа, 12 часов мероприятия текущего контроля успеваемости и промежуточной аттестации), 64 (62\*) часа составляет самостоятельная работа студента.

*\* - если специальный курс читается в нечетном семестре (продолжительность нечетного семестра 18 недель, четного семестра 17 недель).*

7. Входные требования для освоения дисциплины (модуля), предварительные условия.

Для того чтобы изучение дисциплины было возможно, обучающийся должен

1. освоить следующие дисциплины образовательной программы: математический анализ, линейную алгебру, теорию вероятностей, случайные процессы, математическую статистику, английский язык;
2. обладать следующими компетенциями:

Знать: основные направления, проблемы, теории и методы современной математики, основные термины математики, статистики и теории вероятностей на английском языке.

1. Уметь: решать стандартные задачи математического анализа, линейной алгебры, теории вероятностей, случайных процессов, математической статистики и применять идеи, использованные в их решениях, для решения аналогичных задач, читать и переводить на русский язык математическую литературу на английском языке, умень излагать математические рассуждения на английском языке.

Владеть: основными понятиями и теоремами из этих разделов математики.

8. Формат обучения.

очная форма обучения, лекционные занятия.

9. Содержание дисциплины (модуля), структурированное по темам (Перечень тем см. Приложения).

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Наименование и краткое содержание разделов и тем дисциплины (модуля),**  **форма промежуточной аттестации по дисциплине (модулю)** | **Всего**  **(часы**) | В том числе | | | | | | | | |
| **Контактная работа (работа во взаимодействии с преподавателем), часы**  из них | | | | | | **Самостоятельная работа обучающегося, часы**  из них | | |
| Занятия лекционного типа | Занятия семинарского типа | Групповые консультации | Индивидуальные консультации | Учебные занятия, направленные на проведение текущего контроля успеваемости, промежуточной аттестации | **Всего** | Выполнение домашних заданий | Подготовка рефератовит.п.. | **Всего** |
| Тема 1 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 2 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 3 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 4 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 5 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 6 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 7 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 8 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Текущий контроль успеваемости | 6 |  |  |  |  | 2 | 2 | 4 |  | 4 |
| Тема 9 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 10 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 11 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 12 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 13 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 14 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 15 | 6 | 2 |  |  |  |  | 2 | 4 |  | 4 |
| Тема 16 | 4 |  |  |  |  |  | 0 | 4 |  | 4 |
| Тема 17 | 2 |  |  |  |  |  |  | 2 |  | 2 |
| Промежуточная аттестация  *экзамен* | 8 (6\*) |  |  |  |  | 2 | 2 | 6(4\*) |  | 6 (4\*) |
| **Итого** | 108 | 30 |  |  |  | 4 | 34 | 74 |  | 74 |

10. Перечень учебно-методического обеспечения для самостоятельной работы студентов по дисциплине (модулю):

Конспекты лекций, списки задач к лекциям, основная и дополнительная учебная литература.

11. Фонд оценочных средств для промежуточной аттестации по дисциплине (модулю).

* Перечень компетенций:
* Описание шкал оценивания*:*

*экзамен с оценкой по пятибалльной шкале*

* Критерии и процедуры оценивания результатов обучения по дисциплине (модулю), характеризующих этапы формирования компетенций.
* Типовые контрольные задания или иные материалы, необходимые для оценки результатов обучения, характеризующих этапы формирования компетенций.См. Приложения.

12. Ресурсное обеспечение:

Перечень основной учебной литературы: см. Приложение

Перечень дополнительной учебной литературы: см. Приложения

Переченьресурсовинформационно-телекоммуникационнойсети «Интернет»: см. Приложения.

Описание материально-технической базы: аудитория для проведения лекционных занятий, оборудованный интерактивной доской (например, Hitachi Starboard FX-Trio-77E), монитором с диагональю 150 см (минимум), проектором и проекционным экраном; ноутбук; кабинет для подготовки к лекциям, проведения консультаций, хранения учебных материалов; принтер; бумага, картриджи к принтеру; книги и журналы, рекомендованные для курса, в библиотеке, электронный англо-русский словарь Abby Lingvo.

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13. Язык преподавания: английский.

ПРИЛОЖЕНИЕ

1. Insurance Risk Models (МОДЕЛИ СТРАХОВЫХ РИСКОВ).
2. Lecturer – Professor Gennady Falin
3. Module Summary: This module aims provide a grounding in insurance risk models and their simple applications. Many theoretical concepts are introduced through solving of carefully selected problems from the past professional actuarial exams. Indicative Content: loss random variable, individual claims, claim frequency, the principle of equivalence, net premium, individual risk model, the ruin probability, loaded premium, principles of premium calculation, optimal pricing, collective risk model, its relation with the individual risk model, numerical methods.
4. Syllabus:

|  |  |
| --- | --- |
| Unit 1 | Basic terms (insurance contract, premium, etc). Loss random variable, claim frequency and claim severity. Claim settlement pattern. The importance of theoretical distributions in general insurance. Suggested reading: Bowers-section 2.2; Hossack, Pollard, Zehnwirth – sections7.1, 7.3, 7.4, 5.10. |
| Unit 2 | Models for claim frequency: the binomial distribution, the Poisson distribution (as the limit of the binomial distribution), the negative binomial distribution. Heterogeneity of risk. Suggested reading: Panjer, Willmot – section 4.1-4.8, Hossack, Pollard, Zehnwirth – sections 5.6-5.9. |
| Unit 3 | Models for which the claims frequency distribution is a compound distribution: the (a,b) class, compound Poisson claim frequency, Neyman type a distribution, generalized Poisson-Pascal, one parameter primary and modified distributions. Suggested reading: Panjer, Willmot – sections 7.1-7.6 |
| Unit 4 | The Poisson process and its basic properties, the nonhomogeneous Poisson process, a mixed Poisson process, birth process. Suggested reading: Panjer, Willmot – sections 3.3-3.6. |
| Unit 5 | Exposure, “the eighths rule”, “the twenty-fourths rule”. Suggested reading: Hossack, Pollard, Zehnwirth – sections7.2. |
| Unit 6 | Models for claim severity: the uniform distribution, the exponential distribution, the Gamma distribution, the Pareto distribution, the log-normal distribution. Suggested reading: Panjer, Willmot – section 4.9-4.12; Hossack, Pollard, Zehnwirth – sections 5.3,5.4, 5.5. |
| Unit 7 | Coverage limitations (deductibles, retention), inflation, leveraging. Suggested reading: Hogg, Klugman – sections 1.2, 1.3, 5.2A. 5.3, 5.4A, 5.5A, 5.6A; Hossack, Pollard, Zehnwirth – section 7.5. |
| Unit 8 | Inferences from general insurance data: hypothesis testing, point estimations and method of moments, confidence intervals. Risk factors, least squares. Suggested reading: Hossack, Pollard, Zehnwirth-chapter 6. |
| Unit 9 | Individual risk model. Evaluation of the distribution of total claims (convolution, transforms, recursive calculation in the case of life insurance). Approximations to the distribution of total claims. Probability of ruin. Value at risk. Security loading. Reinsurance. Numerical illustrations. Suggested reading: Panjer, Willmot – chapter 5, Bowers-sections 2.1, 2.3-2.5. |
| Unit 10 | The basic premium principles: the expected value principle, the variance principle, the standard deviation principle. A review of other principles. Suggested reading: Virginia R. Young. |
| Unit 11 | Desirable properties of premium principles. Four characterizations of the exponential and the net premium principle. Reduction of premiums through cooperation. The need for reinsurance. Suggested reading: Gerber, chapter 5. |
| Unit 12 | An introduction to risk measures for actuarial applications: risk measures for capital requirements, coherence, measures of variability. estimating risk measures using Monte Carlo simulation. Suggested reading: Hardy. |
| Unit 13 | The optimal pricing of a heterogeneous portfolio. Suggested reading: Falin. |
| Unit 14 | The collective risk model (the Poisson model, the negative binomial model). Analytical results for certain claim size distributions. Limit theorems and approximations. Suggested reading: Bowers-sections 11.1, 11.2,11.3, 11.5, Appendix to chaper 11; Panjer, Willmot – sections 6.1,6.2, 6.12 |
| Unit 15 | Combining/decomposing compound Poisson risks. Suggested reading: Panjer, Willmot – sections 6.3,6.4, Bowers-section 11.4 |
| Unit 16 | Approximating the individual risk model by the compound Poisson model. Suggested reading: Panjer, Willmot – section 6.5. |
| Unit 17 | Computation for arithmetic severities, modifications. Compound negative binomial as a mixture/compound Poisson with multiple claims/contagion model. Suggested reading: Panjer, Willmot – sections 6.6, 6.16, 6.7, 6.8, 6.9. |

1. Типовые контрольные задания или иные материалы, необходимые для оценки результатов обучения, характеризующих этапы формирования компетенций.

Examination questions:

1. Basic terms (insurance contract, premium, etc).
2. Loss random variable, claim frequency and claim severity.
3. Claim settlement pattern.
4. The importance of theoretical distributions in general insurance.
5. Models for claim frequency: the binomial distribution.
6. Models for claim frequency: the Poisson distribution (as the limit of the binomial distribution).
7. Models for claim frequency: the negative binomial distribution. Heterogeneity of risk.
8. Models for which the claims frequency distribution is a compound distribution: the (a,b) class.
9. Models for which the claims frequency distribution is a compound distribution: compound Poisson claim frequency.
10. Models for which the claims frequency distribution is a compound distribution: Neyman type a distribution.
11. Models for which the claims frequency distribution is a compound distribution: generalized Poisson-Pascal, one parameter primary and modified distributions.
12. The Poisson process and its basic properties.
13. The nonhomogeneous Poisson process.
14. A mixed Poisson process.
15. The birth process.
16. Exposure, “the eighths rule”, “the twenty-fourths rule”.
17. Models for claim severity: the uniform distribution, the exponential distribution.
18. Models for claim severity: the Gamma distribution.
19. Models for claim severity: the Pareto distribution.
20. Models for claim severity: the log-normal distribution.
21. Coverage limitations (deductibles, retention).
22. Inflation, leveraging.
23. Inferences from general insurance data: hypothesis testing.
24. Inferences from general insurance data: point estimations and method of moments.
25. Inferences from general insurance data: confidence intervals.
26. Risk factors, least squares.
27. Individual risk model.
28. Evaluation of the distribution of total claims (convolution, transforms, recursive calculation in the case of life insurance).
29. Approximations to the distribution of total claims.
30. Probability of ruin. Value at risk.
31. Security loading.
32. Reinsurance.
33. The basic premium principles: the expected value principle, the variance principle, the standard deviation principle.
34. A review of other premium principles.
35. Desirable properties of premium principles.
36. Four characterizations of the exponential and the net premium principle.
37. Reduction of premiums through cooperation. The need for reinsurance.
38. An introduction to risk measures for actuarial applications: risk measures for capital requirements, coherence, measures of variability. estimating risk measures using Monte Carlo simulation.
39. The optimal pricing of a heterogeneous portfolio.
40. The collective risk model (the Poisson model, the negative binomial model).
41. Analytical results for certain claim size distributions in the collective risk model.
42. Limit theorems and approximations for the collective risk model.
43. Combining/decomposing compound Poisson risks.
44. Approximating the individual risk model by the compound Poisson model.
45. Computation for the collective risk model (arithmetic severities).
46. Compound negative binomial as a mixture/compound Poisson with multiple claims/contagion model.

*Exam papers consist of two questions from the above list and one problem (sample problems are given below).*

Sample Problems:

**Problem 1** (Course 3 -- Actuarial Models, The Society of Actuaries and the Casualty Actuarial Society, November 2000, problem No.21) A claim severity distribution is exponential with mean 1000. An insurance company will pay the amount of each claim in excess of a deductible of 100. Calculate the variance of the amount paid by the insurance company for one claim, including the possibility that the amount paid is 0.

**Problem 2** (Course 1 -- Mathematical Foundations of Actuarial Science, The Society of Actuaries and the Casualty Actuarial Society, November 2000, problem No.25) A manufacturer's annual losses, , (in millions) follow a distribution with density function



To cover its losses, the manufacturer purchases an insurance policy with an annual deductible of . What is the mean of the manufacturer's annual losses not paid by the insurance policy?

**Problem 3** (Course 3 -- Actuarial Models, The Society of Actuaries and the Casualty Actuarial Society, May 2000, problem No.25) An insurance agent will receive a bonus if his loss ratio is less than 70%. You are given:

1. His loss ratio is calculated as incurred losses divided by earned premium on his block of business.
2. The agent will receive a percentage of earned premium equal to 1/3 of the difference between 70% and his loss ratio.
3. The agent receives no bonus if his loss ratio is greater than 70%.
4. His earned premium is 500,000.
5. His incurred losses are distributed according to the Pareto distribution: 

Calculate the expected value of his bonus.

**Problem 4** (The Institute of Actuaries, Exam CT3, September 2009, Problem 6) A random sample of size *n* is taken from an exponential distribution with parameter , that is, with probability density function 

(i) Determine the maximum likelihood estimator (MLE) of .

Claim sizes for certain policies are modelled using an exponential distribution with parameter . A random sample of such claims results in the value of the MLE of  as . A large claim is defined as one greater than  and the claims manager is particularly interested in *p*, the probability that a claim is a large claim.

(ii) Determine , the MLE of *p*, explaining why it is the MLE.

**Problem 5** (The Institute of Actuaries, Exam CT3, September 2009, Problem 9 -- a revised version) In a group of motor insurance policies issued by a company, 80% of claims are made on comprehensive policies and 20% are made on third-party-only policies. An amount paid out by the company on a comprehensive policy claim has mean value  and the standard deviation , and an amount paid out on a third-party-only policy claim has mean value  and the standard deviation .

* Calculate the mean value and the variance of the amount paid out an a claim.
* Given that the total number of policies is  and, on average, the claim rate is  claims per policy per year calculate:

1. the total expected amount paid out in claims by the company in one year,
2. the standard deviation of the amount paid out in claims by the company in one year.

**Problem 6.** An insurance company covers *n*=2 buildings against fire damage up to an amount stated in the policy. For the first building the sum insured is  (million pounds), and for the second is  (million pounds). The fires in the buildings are independent events. The probability of a fire in a one-year period is  for the first building and  for the second building. For th building, the fire damage  (given that the fire has happened) is uniformly distributed over interval . Calculate the value at risk as a function of the ruin probability acceptable for the company.

**Problem 7** (Course 1 -- Mathematical Foundations of Actuarial Science, The Society of Actuaries and the Casualty Actuarial Society, November 2000, problem No.36.) An insurance company insures a large number of drivers. Let *X* be the random variable representing the company's losses under collision insurance, and let *Y* represent the company's losses under liability insurance. *X* and *Y* have joint density function



What is the probability that the total loss is at least 1?

**Problem 8** (The Institute of Actuaries, Exam CT6, April 2008, Problem 10) A bicycle wheel manufacturer claims that its products are virtually indestructible in accidents and therefore offers a guarantee to purchasers of pairs of its wheels. There are 250 bicycles covered, each of which has a probability *p* of being involved in an accident (independently). Despite the manufacturer's publicity, if a bicycle is involved in an accident, there is in fact a probability of 0.1 for each wheel (independently) that the wheel will need to be replaced at a cost of . Let *S* denote the total cost of replacement

wheels in a year.

* Show that the moment generating function of *S* is given by ;
* Show that  and .

Suppose instead that the manufacturer models the cost of replacement wheels as a random variable *T* based on a portfolio of 500 wheels, each of which (independently) has a probability of $0.1p$ of requiring replacement.

* Derive expressions for  and  in terms of *p*.
* Suppose *p* = 0.05.

1. Calculate the mean and variance of *S* and T.
2. Calculate the probabilities that *S* and *T* exceed .
3. Comment on the differences.

**Problem 9** (The Institute of Actuaries, Exam CT3, April 2006, Problem 8) The events that lead to potential claims on a policy arise as a Poisson process at a rate of $\lambda=0.8$ per year. However the policy is limited such that only the first three claims in any one year are paid.

* Determine the probabilities of 0, 1, 2 and 3 claims being paid in a particular year.
* The amounts (in units of ) for the claims paid follow a gamma distribution with parameters  and . Calculate the expectation of the sum of the amounts for the claims paid in a particular year.
* Calculate the expectation of the sum of the amounts for the claims paid in a particular year, given that there is at least one claim paid in the year.

**Problem 10** (The Institute of Actuaries, Exam CT6, September 2009, Problem 10) The total number of claims *N* on a portfolio of insurance policies has a Poisson distribution with mean . Individual claim amounts are independent of  and each other, and follow a distribution  with mean  and variance . The total aggregate claims in the year is denoted by *S*. The random variable *S* therefore has a compound Poisson distribution.

(i) Derive an expression for the moment generating function of *S* in terms of the moment generating function of *X*.

(ii) Derive expressions for the mean and variance of *S* in terms of ,  and .

For a particular type of policy, individual losses are exponentially distributed with mean 100. For losses above 200 the insurer incurs an additional expense of 50 per claim.

(iii) Calculate the mean and variance of *S* for a portfolio of such policies with .

**Problem 11** (The Institute of Actuaries, Exam CT6, April 2007, Problem 7 -- a revised version) The total claims arising from a certain portfolio of insurance policies over a given month is represented by , where

* *N* has a Poisson distribution with mean 2,
*  is a sequence of independent random variables that are also independent of *N* and have a common distribution , .

An aggregate reinsurance contract has been arranged such that the amount paid by the reinsurer is , if , and zero otherwise. The aggregate claims paid by the direct insurer and the reinsurer are denoted by  and , respectively. Calculate  and .

**Problem 12** (The Institute of Actuaries, Exam CT6, April 2009, Problem 9) Individual claims under a certain type of insurance policy are for either 1 (with probability ) or 2 (with probability ). The insurer is considering entering into an excess of loss reinsurance arrangement with retention 

(where ). Let  denote the amount paid by the insurer (net of reinsurance) on the th claim.

* Calculate and simplify expressions for the mean and variance of  .

Now assume that . The number of claims in a year follows a Poisson distribution with mean 500. The insurer wishes to set the retention so that the probability that aggregate claims in a year will exceed 700 is less than 1%.

* Show that setting  gives the desired result for the insurer.

**Problem 13** (The Institute of Actuaries, Exam CT6, September 2008, Problem 11) Losses on a portfolio of insurance policies in 2006 are assumed to have an exponential distribution with parameter . In 2007 loss amounts have increased by a factor *k* (so that a loss incurred in 2007 is k times an equivalent loss incurred in 2006).

(i) Show that the distribution of loss amounts in 2007 is also exponential and determine the parameter of the distribution.

Over the calendar years 2006 and 2007 the insurer had in place an individual excess-of-loss reinsurance arrangement with a retention of *M*. Claims paid by the insurer were:

2006: 4 amounts of *M* and 10 claims under *M* for a total of 13,500.

2007: 6 amounts of *M* and 12 claims under *M* for a total of 17,000.

(ii) Show that the maximum likelihood estimate of  is .

(iii) The insurer is negotiating a new reinsurance arrangement for 2008. The retention was set at 1600 when the current arrangement was put in place in 2006. Loss inflation between 2006 and 2007 was 10% (i.e. *k* = 1.1) and further loss inflation of 5% is expected between 2007 and 2008.

(a) Use this information to calculate .

(b) The insurer wishes to set the retention ** for 2008 such that the expected (net of re-insurance) payment per claim for 2008 is the same as the expected payment per claim for 2006. Calculate the value of ** , using your estimate of  from (iii)(a).

**Problem 14** (The Institute of Actuaries, Exam CT6, April 2006, Problem 10) An insurance company has two portfolios of independent policies, on each of which claims occur according to a Poisson process. For the first portfolio, all claims are for a fixed amount of  and 10 claims are expected per annum. For the second portfolio, claim amounts are exponentially distributed with mean  and 30 claims are expected per annum. Let *S* denote aggregate annual claims from the two portfolios together. A check is made for ruin only at the end of the year. The insurer includes a loading of 10% in the premiums, for all policies.

(i) Calculate the mean and variance of *S*.

(ii) Use a normal approximation to the distribution of *S* to calculate the initial capital, *u*, required in order that the probability of ruin at the end of the first year is 0.01.

The insurer is considering purchasing proportional reinsurance from a reinsurer that includes a loading of  in its premiums. The proportion of each claim to be retained by the direct insurer is  (). Let  denote the aggregate annual claims paid by the direct insurer on the two portfolios

together, net of reinsurance.

(iii)] Use a normal approximation to the distribution of  to show that the initial capital, ** , required in order that the probability of ruin at the end of the first year is 0.01 can be written as 

(iv) Show that  , as long as .

(v) Show that  decreases as  increases, and discuss the practical implications of this result.

Sample examination papers.

**Paper 1**

**1.** Loss random variable, claim frequency and claim severity.

**2.** The basic premium principles: the expected value principle, the variance principle, the standard deviation principle.

**Problem.** The events that lead to potential claims on a policy arise as a Poisson process at a rate of $\lambda=0.8$ per year. However the policy is limited such that only the first three claims in any one year are paid.

* Determine the probabilities of 0, 1, 2 and 3 claims being paid in a particular year.
* The amounts (in units of ) for the claims paid follow a gamma distribution with parameters  and . Calculate the expectation of the sum of the amounts for the claims paid in a particular year.
* Calculate the expectation of the sum of the amounts for the claims paid in a particular year, given that there is at least one claim paid in the year.

**Paper 2**

**1.** Models for claim severity: the log-normal distribution.

**2.** Coverage limitations (deductibles, retention).

**Problem.** Individual claims under a certain type of insurance policy are for either 1 (with probability ) or 2 (with probability ). The insurer is considering entering into an excess of loss reinsurance arrangement with retention 

(where ). Let  denote the amount paid by the insurer (net of reinsurance) on the th claim.

* Calculate and simplify expressions for the mean and variance of  .

Now assume that . The number of claims in a year follows a Poisson distribution with mean 500. The insurer wishes to set the retention so that the probability that aggregate claims in a year will exceed 700 is less than 1%.

* Show that setting  gives the desired result for the insurer.

**Paper 3**

**1.** Inflation, leveraging.

**2.** Limit theorems and approximations for the collective risk model

**Problem.** An insurance agent will receive a bonus if his loss ratio is less than 70%. You are given:

1. His loss ratio is calculated as incurred losses divided by earned premium on his block of business.
2. The agent will receive a percentage of earned premium equal to 1/3 of the difference between 70% and his loss ratio.
3. The agent receives no bonus if his loss ratio is greater than 70%.
4. His earned premium is 500,000.
5. His incurred losses are distributed according to the Pareto distribution: 

Calculate the expected value of his bonus.

1. Suggested reading. Перечень основной и дополнительной учебной литературы, ресурсов информационно-телекоммуникационной сети «Интернет»:

**Core Texts**

1. Harry H.Panjer, Gordon E.Willmot. Insurance Risk Models. The Society of Actuaries, 1992. ISBN 0-938959-25-5

2. Robert V.Hogg, Stuart A.Klugman. Loss Distributions. John Wiley & Sons, 1984.ISBN 0-471-87929-0

3. I.B.Hossack, J.H.Pollard, B.Zehnwirth. Introductory Statistics with Applications in General Insurance. Cambridge University Press, 1989. ISBN 0-521-28957-2/ 0-521-24781-0

4. Hans U.Gerber. An Introduction to Mathematical Risk Theory. Wharton School, University of Pennsylvania, 1979.

5. Newton l.Bowers, Hans U.Gerber, et al. Actuarial Mathematics. The Society of Actuaries. 1997.

6. D.G.Hart, R.A.Buchanan, B.A.Howe. The Actuarial Practice of General Insurance. Institute of Actuaries of Australia, Sydney, 1996. ISBN 0-85813-055-6

7. G.Falin. On the optimal pricing of a heterogeneous portfolio. *ASTIN Bulletin,*  2008, vol.38, #1, 161-170.

8. Virginia R. Young. Premium Principles. Encyclopedia of Actuarial Science. John Wiley & Sons, 2004.

9. Mary R. Hardy. An Introduction to Risk Measures for Actuarial Applications. The Casualty Actuarial Society and the Society of Actuaries, 2006.

**Recommended Reading**

1. Г.И.Фалин. *Математический анализ рисков в страховании,* Российский юридический издательский дом, Москва, 1994. ISBN 5-88635-003-0

2. Г.И.Фалин, А.И.Фалин.*Теория риска для актуариев в задачах*, 2-е издание: Мир, Москва, 2004. 240 c., ил. ISBN 5-03-003607-5

Перечень ресурсов информационно-телекоммуникационной сети «Интернет»:

<http://www.actuaries.org.uk/>

<https://www.soa.org/member/>

<http://cbr.ru/finmarkets/?PrtId=supervision_actuary>

<http://www.journals.elsevier.com/insurance-mathematics-and-economics/>

<http://journalofriskandinsurance.smeal.psu.edu/>