Discounting in The New World

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1 Black-Scholes PDE (in the old world)

- Risk-free money market account with risk free rate r
- Stock price model

$$dS = \mu S\, dt + \sigma S\, dW.$$

- Option $V(S_t, t)$
- Ito's lemma for option

$$dV = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \frac{\partial V}{\partial S} dS$$

- Replicate over $[t, t+dt]$ with stock and cash. Portfolio

$$\Pi = \Delta S + \beta$$

  - Stock position: $\Delta = \frac{\partial V}{\partial S}$
  - Cash position: self-financing

$$dV = d\Pi = \Delta dS + r\beta dt$$

- Hence cash position is

$$\beta = \frac{1}{r} \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right)$$

2 Black-Scholes PDE

- Make replicating portfolio agree with the option on the terminal date,

$$V\left(S_T, T\right) = \Pi_T$$

- From self-financing, get

$$\begin{aligned} V\left(S_t, t\right) &= \Pi_t = \Delta S + \beta \\ &= S\frac{\partial V}{\partial S} + \frac{1}{r}\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}\right) \end{aligned}$$

- Re-arranging, obtain

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV.$$

- Would have obtained if started from an SDE

$$\begin{aligned} dS &= rS\,dt + \sigma S\,dW^Q, \\ V\left(S_t, t\right) &= \mathrm{E}^Q\left(e^{-\int_t^T r\,ds} V\left(S_T, T\right)\Big|\, t, S_t\right). \end{aligned}$$

  Here Q is *risk-neutral* probability.

- Do not need to know μ.

3 What is wrong with Black Scholes

- Where is that credit-risk-free money market account?
  - Give cash to another bank?
  - Give cash to a government?
- Nothing in modern economy looks like a classic money market account
- How to build an asset pricing theory without risk free rate?
- Asset pricing theory traditionally starts with assets that pay no dividends and have a payoff at maturity. Is that how assets in modern economy look like?

4 Credit risk mitigation in OTC trading

- Over-the-counter (bilateral) trading is governed by legal documents, primary of which is ISDA Master Agreement
- Part of it, Credit Support Annex (CSA) specifies credit risk mitigation in form of collateral posting
- In broad strokes, it specifies that if party A owes money to party B, it has to post collateral in that amount, and vice versa
- So if A defaults, B could take that collateral in lieu of the promise of A
- CSA specifies other important credit risk mitigants such as netting – if A owes B on one contract and B owes A on some other, they can be offset against each other in the case of default (different from traditional bankruptcy law but it is a different story)
- CSAs between each two parties are (somewhat) different. CSA specifies
  - Eligible collateral (cash in a number of currencies, bonds)
  - Rates paid on collateral (party holding collateral typically pays certain rate to the collateral "owner")
  - frequency of collateral posting (e.g. daily)
  - Threshold

5 Collateralized Assets

- Let us look at the mechanics of collateralized trading
- Party A sells a call option to party B
- B pays $V(0)$ dollars to A
- A promises to pay the payoff of the option at expiry to B
- Any promise needs to be collateralized. A needs to post collateral. How much?
- Well, it is the value of the promise (option) so $V(0)$ dollars! They go right back to B
- During life, the value of the option fluctuates. Depending on the move A will post or claim back collateral
- B will pay an agreed-upon overnight rate on the outstanding collateral to A
- At any point in time the t total collateral posted by A will be $V(t)$ which is the value of the option on that day

6 Collateralized Assets

- Note that at any time the option contract could be dissolved and collateral kept – the collateral will exactly offset the market value of the option
- In particular, at option expiry B will just keep the collateral it has and A does not need to pay anything else
- Quite different from a classic picture
- Details in [Pit12]

7 Hedging Instruments

- Trading in hedging "cash" instruments (stocks, bonds) fits the same pattern
- When we need to buy stock, where does the bank get money? (How does it fund the shares)
- By borrowing them, with the borrow secured by the shares just bough!
- This is called a repo transaction
- The rate for this loan is the repo rate
- Borrow the money, buy stock
- Deliver shares as collateral for the loan
- Get collateral back the next day
- Return the loan and the overnight interest (repo rate)
- Repeat for as many days as the shares are needed
- Paying repo rate more efficient than borrowing unsecured – lower rate due to absence of credit risk

8 Zero-Price Dividend-Paying Assets

- Traditional APT
  - Starts with dividend-free assets and money market account
  - Dividend-paying assets are incorporated by reinvesting dividends into the asset itself
  - Some dividend-paying assets have zero price: futures
  - Reinvestment approach fails for futures but as long as there is a money market account can cover by ATP
- Collateralized assets (and hedging instruments) are *zero-price dividend-paying assets* (ZPDP)
  - Can be entered into and exited at zero cost
  - Pay dividend (in the form of collateral rate) continuously
- The only credit-risk-free assets in modern economy are ZPDP assets
  - And there is no money-market account!
  - And none of the assets can serve as a numeraire (zero price)
- Need APT built from these assets

9 ZPDP Assets and No Arbitrage

- A ZPDP asset is an asset that
  - Can be "bought" for no money at any time
  - Gives the holder the right to a dividend stream until the asset is sold
  - Can be "sold" at any time for no money
- Economy is modelled by the p-dimensional *cumulative-dividend process* $G(t) = (G_1(t), \ldots, G_p(t))^\top$ of zero-price assets
  - $G_i(t)$ is the total dividend paid by the asset i over the time period $[0, t]$
- Trading strategy is a predictable adapted process $\phi(t) = (\phi_1(t), \ldots, \phi_p(t))^\top$ where ϕ_i is holding of the i-th asset at time t
  - For convenience only consider trading strategies that are identically zero after some time, i.e. there exists T such that $\phi(t) \equiv 0$ for $t \geq T$

10 ZPDP Assets and No Arbitrage

- Total gains H^ϕ on this strategy are given by

$$H^\phi = \int_0^\infty \phi(t)^\top \, dG(t)$$

- As the cost of entering or exiting any position is always zero, the arbitrage opportunity in this economy is defined as the existence of the strategy ϕ such that

$$H^\phi \geq 0 \text{ a.s., and } \mathrm{P}\left(H^\phi > 0\right) > 0$$

- Main result (see [AP16]): if the economy admits no arbitrage, then there exists an equivalent martingale measure Q such that G is a Q-martingale
  - Probably covered by some very general flavor of the Fundamental Theorem of Asset Pricing but interesting to look in detail at this special case

11 Proof of the Main Result

- Let G be given, under P, by

$$dG(t) = \mu(t)\,dt + \sigma(t)\,dW(t)$$

  - $W(t)$ a d-dimensional Brownian motion
  - $\mu(t) = \mu(t, \omega)$ is p-dimensional and $\sigma(t) = \sigma(t, \omega)$ is $p \times d$-dimensional

- The total gains for any strategy ϕ are given by

$$H^{\phi} = \int_0^{\infty} \phi(t)^{\top} \mu(t)\,dt + \int_0^{\infty} \phi(t)^{\top} \sigma(t)\,dW(t)$$

- First a simpler case of $\sigma(t)$ having rank p ($d \geq p$). We can find a d-dimensional vector $\theta(t)$ such that

$$\mu(t) = \sigma(t)\theta(t),$$

  so that

$$dG(t) = \sigma(t)\theta(t)\,dt + \sigma(t)\,dW(t) = \sigma(t)\,\left(dW(t) + \theta(t)\,dt\right)$$

- The measure Q is then given by Girsanov's theorem – it is the measure under which $dW(t) + \theta(t)\,dt$ is the driftless Brownian motion

12 Proof of the Main Result

- More interesting case of $\sigma(t)$ with rank strictly less than p. Then there exists a p-dimensional vector $\overline{\phi} \neq 0$ such that

$$\overline{\phi}^\top \sigma(t) \equiv 0 \text{ a.s.} \tag{1}$$

- Trading strategy:

$$\phi(s) = 1_{\{t \leq s \leq t+dt\}} \left(\overline{\phi} 1_{\left\{\overline{\phi}^\top \mu(t) > 0\right\}} - \overline{\phi} 1_{\left\{\overline{\phi}^\top \mu(t) \leq 0\right\}} \right)$$

- Total gains on this strategy

$$\begin{aligned} H^\phi &= \int_0^\infty \phi(s)^\top \mu(s)\, dt + \int_0^\infty \phi(s)^\top \sigma(s)\, dW(s) \\ &= \overline{\phi}^\top \mu(t) \left(1_{\left\{\overline{\phi}^\top \mu(t) > 0\right\}} - 1_{\left\{\overline{\phi}^\top \mu(t) < 0\right\}} \right) dt + 0 \\ &= \left| \overline{\phi}^\top \mu(t) \right| dt \end{aligned}$$

- To ensure no-arbitrage

$$\overline{\phi}^\top \mu(t) = 0 \text{ a.s.} \tag{2}$$

13 Proof of the Main Result

- We have shown so far that for any vector $\overline{\phi}$, $\overline{\phi}^{\top}\sigma(t) = 0$ implies $\overline{\phi}^{\top}\mu(t) = 0$.
- Therefore, $\mu(t)$ is in the range of $\sigma(t)$ and there exists a d-dimensional vector $\theta(t)$ such that

$$\mu(t) = \sigma(t)\theta(t)$$

- The rest of the argument follows the rank-p case above:

$$dG(t) = \sigma(t)\theta(t)\,dt + \sigma(t)\,dW(t) = \sigma(t)\,\left(dW(t) + \theta(t)\,dt\right)$$

- The measure Q is not associated with any particular numeraire, unlike in the tradition APT
- Not much of a problem as they work just as well as the "traditional" ones

Notations

- $V(t)$ is price of a collateralized asset between party A and B. If $V(t) > 0$ for A, party B will post $V(t)$ to A.
- $c(t)$ is a contractually specified collateral rate $c(t)$ on $V(t)$. If $V(t) > 0$, A will pay this rate to B

15 Collateralized Cashflow Analysis

Assume A "buys" some collateralized asset from B

1. Purchase of the asset. The amount of $V(t)$ is paid by A to B
2. Collateral at t. Since A's mark-to-market is $V(t)$, the amount $V(t)$ of collateral is posted by B to A
3. Return of collateral. At time $t + dt$ A returns collateral $V(t)$ to B
4. Interest. At time $t + dt$, A also pays $V(t)c(t)\,dt$ interest to B
5. New collateral. The new mark-to-market is $V(t+dt)$. Party B pays $V(t+dt)$ in collateral to A.

Note that there is no actual cash exchange at time t. At time $t + dt$, net cash flow to A is given by

$$V(t + dt) - V(t)(1 + c(t)\,dt) = dV(t) - c(t)V(t)\,dt.$$

As already noted, at time $t + dt$, the MTM+collateral for each party is 0, meaning they can terminate the contract (and keep the collateral) at no cost

- *Collateralized asset is a ZPDP asset*

16 Valuation Formula

- Economy with p collateralized derivatives, some may be stocks or bonds with attached repo agreements
- Value processes $V_1(t), \ldots, V_p(t)$, collateral rates $c_1(t), \ldots, c_p(t)$, cumulative-dividend processes $G_i(t)$, $i = 1, \ldots, p$
- It follows from the previous slide that

$$dG_i(t) = dV_i(t) - c_i(t)V_i(t)\,dt,\; i = 1, \ldots, p$$

- Express V_i in terms of G_i:

$$\begin{aligned} d\left(e^{-\int_0^t c_i(s)\,ds} V_i(t)\right) &= -c_i(t)e^{-\int_0^t c_i(s)\,ds} V_i(t)\,dt + e^{-\int_0^t c_i(s)\,ds} dV_i(t) \\ &= -c_i(t)e^{\int_0^t c_i(s)\,ds} V_i(t)\,dt + e^{-\int_0^t c_i(s)\,ds}\left(dG_i(t) + c_i(t)V_i(t)\,dt\right) \\ &= e^{-\int_0^t c_i(s)\,ds} dG_i(t) \end{aligned}$$

and, for any $t \leq T$,

$$e^{-\int_0^T c_i(s)\,ds} V_i(T) - e^{-\int_0^t c_i(s)\,ds} V_i(t) = \int_t^T e^{-\int_0^u c_i(s)\,ds} dG_i(u) \qquad (3)$$

17 Valuation Formula

- By the main result there exists a risk-neutral measure Q in which all $G_i(t)$, $i = 1, \ldots, p$, are martingales.
- Applying $\mathrm{E}_t^{\mathrm{Q}}$ to (3):

$$e^{-\int_0^T c_i(s)\,ds} V_i(T) - e^{-\int_0^t c_i(s)\,ds} V_i(t) = \int_t^T e^{-\int_0^u c_i(s)\,ds} dG_i(u)$$

  and using the martingale property gives us the main valuation formula for collateralized derivatives,

$$V_i(t) = \mathrm{E}_t^{\mathrm{Q}} \left(e^{-\int_t^T c_i(s)\,ds} V_i(T) \right), \; i = 1, \ldots, p \tag{4}$$

- The value at time t of a collateralized derivative is equal to the expectation of its value at a future time $T \geq t$ discounted at its own collateral rate

18 Example: Two Collateralized Assets

- Simple example of the general result: two assets collateralized with rates $c_1(t)$ and $c_2(t)$
- In real world measure the asset prices follow

$$dV_i(t) = \mu_i(t)V_i(t)\,dt + \sigma_i(t)V_i(t)\,dW(t), \quad i = 1, 2. \tag{5}$$

- Note the same Brownian motion. Case of a stock (i.e. a repo transaction with stock) and an option on that stock.
- At time t form a portfolio to hedge the effect of randomness of $dW(t)$ on the cash exchanged at time $t + dt$ (no cash exchange at t)
- Go long asset 1 notional $\sigma_2(t)V_2(t)$ and go short asset 2 notional $\sigma_1(t)V_1(t)$
- The cash exchange at time $t + dt$ is then equal to

$$\begin{aligned} & \sigma_2(t)V_2(t)\,(dV_1(t) - c_1(t)V_1(t)\,dt) - \sigma_1(t)V_1(t)\,(dV_2(t) - c_2(t)V_2(t)\,dt) \\ = \; & \sigma_2(t)V_1(t)V_2(t)\,(\mu_1(t) - c_1(t))\;dt - \sigma_1(t)V_1(t)V_2(t)\,(\mu_2(t) - c_2(t))\;dt \end{aligned}$$

- This amount is known at time t and the contract can be terminated at $t + dt$ at zero cost. Hence, the only way both parties agree to transact on this portfolio (no arbitrage), this cash flow must actually be zero

19 Example: Two Collateralized Assets

- Hence

$$\sigma_2(t)\left(\mu_1(t) - c_1(t)\right) = \sigma_1(t)\left(\mu_2(t) - c_2(t)\right)$$

- Using this we can rewrite (5) as

$$dV_i(t) = c_i(t)V_i(t)\,dt + \sigma_i(t)V_i(t)\,d\tilde{W}(t), \quad i = 1, 2, \tag{6}$$

where

$$d\tilde{W}(t) = dW(t) + \frac{\mu_1(t) - c_1(t)}{\sigma_1(t)}dt = dW(t) + \frac{\mu_2(t) - c_2(t)}{\sigma_2(t)}dt$$

- Now, looking at (6) we see that there exists a measure Q, equivalent to the real world one, in which asset i grows at rate $c_i(t)$.
- In Q, the price process for each asset is given by

$$V_i(t) = \mathrm{E}_t^{\mathrm{Q}}\left(e^{-\int_t^T c_i(s)\,ds}V_i(T)\right), \quad i = 1, 2 \tag{7}$$

20 Domestic and Foreign Collateral

- Many CSAs allow for delivery of cash in different currencies
- We need to consider zero coupon bonds (ZCBs) collateralized in the domestic, and as well as some other (call it foreign) currency
- Economy with domestic and foreign assets and an FX rate $X(t)$ expressed as a number of domestic ($\mathcal{D}$) units per one foreign ($\mathcal{F}$)
- The domestic collateral rate is $c_d(t)$ and the foreign rate is $c_f(t)$
- Domestic ZCB collateralized in domestic currency by $P_{d,d}(t,T)$. This bond generates the following cashflow at time $t+dt$,

$$dG_{d,d}(t,T) = dP_{d,d}(t,T) - c_d(t)P_{d,d}(t,T)\,dt \tag{8}$$

21 Foreign Bonds with Domestic Collateral

- Now consider a foreign ZCB collateralized with the domestic rate. Let its price, in foreign currency, be $P_{f,d}(t,T)$. Cashflows:

1. Purchase of the asset. The amount of $P_{f,d}(t,T)$ is paid (in foreign currency $\mathcal{F}$) by party A to B.
2. Collateral at t. Since A's MTM is $P_{f,d}(t,T)$ in foreign currency, the amount $P_{f,d}(t,T)X(t)$ of collateral is posted in domestic currency $\mathcal{D}$ by B to A
3. Return of collateral. At time $t+dt$ A returns collateral $P_{f,d}(t,T)X(t)\mathcal{D}$ to B
4. Interest. At time $t+dt$, A also pays $c_d(t)P_{f,d}(t,T)X(t)\,dt$ interest to B in $\mathcal{D}$
5. New collateral. The new MTM is $P_{f,d}(t+dt,T)$. Party B pays $P_{f,d}(t+dt,T)X(t+dt)$ collateral to A in $\mathcal{D}$

The cash flow, in $\mathcal{D}$, at $t+dt$ is

$$dG_{f,d}(t,T) = d\left(P_{f,d}(t,T)X(t)\right) - c_d(t)P_{f,d}(t,T)X(t)\,dt \qquad (9)$$

22 Drift of FX Rate

- Equations (8), (9) are insufficient to determine the drift of X
- From (9) we can only deduce the drift of the combined quantity $XP_{f,d}$ and the drift of $P_{f,d}$ is in general *not* c_f (nor it is c_d, for that matter)
- To understand the drift of $X(\cdot)$, we need to understand what kind of (domestic) cash flow we can generate from holding a unit of foreign currency
- Suppose we have $1\mathcal{F}$. If it was a unit of stock, we could repo it out (i.e. borrow money secured by the stock) and pay a repo rate on the stock
- In FX, having $1\mathcal{F}$, we can give it to another dealer and receive its price in domestic currency, $X(t)\mathcal{D}$. The next instant $t+dt$ we would get back $1\mathcal{F}$, and pay back $X(t) + r_{d,f}(t)X(t)dt$, where $r_{d,f}(t)$ is a *rate agreed on this domestic loan collateralized by* $\mathcal{F}$. As we can sell $1\mathcal{F}$ for $X(t+dt)\mathcal{D}$ at time $t+dt$ the cash flow at $t+dt$ would be

$$dG_X(t) = dX(t) - r_{d,f}(t)X(t)\,dt$$

- This is an "instantaneous" (aka tom/next in actual market) FX swap
- Importantly, the rate $r_{d,f}(t)$ has no relationship to collateralization rates in two different currencies

23 Cross-Currency Model under Domestic Collateral

1. Market in instantaneous FX swaps allows us to generate cash flow $dX(t) - r_{d,f}(t)X(t)\,dt$
2. Market in $P_{d,d}$ generates cash flow $dP_{d,d}(t,T) - c_d(t)P_{d,d}(t,T)\,dt$
3. Market in $P_{f,d}$ generates cash flow $d\left(P_{f,d}(t,T)X(t)\right) - c_d(t)P_{f,d}(t,T)X(t)\,dt$

- Assume real world measure dynamics (μ, dW are vectors and Σ is a matrix)

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d\left(P_{f,d}X\right)/\left(P_{f,d}X\right) \end{pmatrix} = \mu\,dt + \Sigma\,dW,$$

- By our main result, we can find a measure ("domestic risk-neutral") $\mathbb{Q}^d$ under which the dynamics are

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d\left(P_{f,d}X\right)/\left(P_{f,d}X\right) \end{pmatrix} = \begin{pmatrix} r_{d,f} \\ c_d \\ c_d \end{pmatrix} dt + \Sigma\,dW^d \qquad (10)$$

With

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d\left(P_{f,d}X\right)/\left(P_{f,d}X\right) \end{pmatrix} = \begin{pmatrix} r_{d,f} \\ c_d \\ c_d \end{pmatrix} dt + \Sigma\, dW^d,$$

we have

$$\begin{aligned} X(t) &= \mathrm{E}_t^d\left(e^{-\int_t^T r_{d,f}(s)\,ds} X(T)\right), \\ P_{d,d}(t,T) &= \mathrm{E}_t^d\left(e^{-\int_t^T c_d(s)\,ds}\right), \\ P_{f,d}(t,T) &= \frac{1}{X(t)}\mathrm{E}_t^d\left(e^{-\int_t^T c_d(s)\,ds} X(T)\right). \end{aligned} \tag{11}$$

25 Cross-Currency Model under Foreign Collateral

- Same model under foreign collateralization
- Foreign bonds $P_{f,f}$ and domestic bonds collateralized in foreign currency $P_{d,f}$
- By repeating the arguments above we can find a measure Q^f under which

$$\begin{pmatrix} d(1/X)/(1/X) \\ dP_{f,f}/P_{f,.f} \\ d\left(P_{d,f}/X\right)/\left(P_{d,f}/X\right) \end{pmatrix} = \begin{pmatrix} -r_{d,f} \\ c_f \\ c_f \end{pmatrix} dt + \tilde{\Sigma}\, dW^f \tag{12}$$

- In particular

$$P_{d,f}(t,T) = X(t)\mathrm{E}_t^f\left(e^{-\int_t^T c_f(s)\,ds}\frac{1}{X(T)}\right). \tag{13}$$

- Not all processes in (10) and (12) can be specified independently. In fact, with the addition of the dynamics of $P_{f,f}$ to (10), the model is fully specified, as the dynamics of $P_{d,f}$ can then be derived

- A forward FX contract pays $X(T) - K$ at T (in $\mathcal{D}$). The price process of the *domestic-currency*-collateralized forward contract is

$$\mathrm{E}_t^d\left(e^{-\int_t^T c_d(s)\,ds}\,(X(T) - K)\right) = X(t)P_{f,d}(t,T) - KP_{d,d}(t,T)$$

- The *forward FX rate*, i.e. K that makes the price process have value zero is given by $X_d(t,T) = \frac{X(t)P_{f,d}(t,T)}{P_{d,d}(t,T)}$.
- We can also view a forward FX contract as paying $1 - K/X(T)$ in $\mathcal{F}$
- Then, with *foreign collateralization*, the value would be

$$\mathrm{E}_t^f\left(e^{-\int_t^T c_f(s)\,ds}\,(1 - K/X(T))\right) = P_{f,f}(t,T) - KP_{d,f}(t,T)/X(t)$$

  and the forward FX rate collateralized in c_f is given by $X_f(t,T) = \frac{X(t)P_{f,f}(t,T)}{P_{d,f}(t,T)}$
- In the general model, there is no reason why $X_f(t,T)$ would be equal to $X_d(t,T)$, and the forward FX rate would depend on the collateral used. It appears, however, that in current market practice FX forwards are quoted without regard for the collateral arrangements

27 Choice Collateral

- Consider a domestic asset, with price process $V(t)$, that can be collateralized either in the domestic (rate c_d) or the foreign (rate c_f) currency.
- Common case for CSA agreements between dealers
- From previous analysis it follows that the foreign-collateralized domestic ZCB grows (in the domestic currency) at the rate $c_f + r_{d,f}$
- It can be shown rigorously that the same is true for any domestic asset
- When one can choose the collateral, one would maximize the rate received on it, so the choice collateral rate is equal to

$$\max(c_d(t), c_f(t) + r_{d,f}(t)) = c_d(t) + \max(c_f(t) + r_{d,f}(t) - c_d(t), 0)$$

- The simplest extension of the traditional cross-currency model that accounts for different collateralization would keep the *collateral basis*

$$q_{d,f}(t) \triangleq c_f(t) + r_{d,f}(t) - c_d(t)$$

deterministic (intrinsic)

28 Choice Collateral

- In this case the collateral choice will not generate any optionality although the discounting curve for the choice collateral rate will be modified
- Anecdotal evidence suggests that at least some dealers do assign some value to the option to switch collateral in the future
- Full collateral choice model:

$$V(t) = \mathrm{E}_t^d \left(e^{-\int_t^T c_d(s)\,ds} e^{-\int_t^T \max(q_{d,f}(s),0)\,ds} V(T) \right)$$

- At least 4 factors: one for each of c_d, c_f, X, $q_{d,f}$. "Standard" XC model recovered with $q_{d,f} \equiv 0$.
- Need to price options even for simplest products!

29 Issues with Full Collateral Choice Model

- Large number of unobserved parameters (volatilities, correlations of $q_{d,f}$)
- Uncertain horizon – collateral choice may go away with developments in the industry (more clearing, standard CSA)
- Assumes that instantaneous replacement of collateral from one currency to another is possible
- More realistic assumptions (?)
  - Only *change* in collateral balance can be posted in a choice currency
  - Only currency previously posted can be recalled, not exceeding the total amount posted (and change in MTM)
- This results in a path-dependent, non-linear dynamic optimization problem
- All in all, swaps pricing is getting quite complicated! More details in [Pit10], [Pit12], [Pit13a], [Pit13b]

30 Barclays Graduate and Intern Program

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