Scatter matrices, principal component analysis (PCA), and independent component analysis (ICA)

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ABSTRACT: Assume that a random p-vector $\boldsymbol{s}$ has independent components and that $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{b}$ where $\boldsymbol{A}$ is a positive definite $p \times p$ matrix and $\boldsymbol{b}$ is a p-vector. Assume that $\boldsymbol{x}_1, ..., \boldsymbol{x}_n$ is a random sample from the distribution F of $\boldsymbol{x}$. In the *independent component analysis (ICA)* one tries to estimate a matrix $\boldsymbol{B}$ such that $\boldsymbol{B}\boldsymbol{x}$ has independent components. On the other hand, let $\boldsymbol{S}(F)$ or $\boldsymbol{S}(\boldsymbol{x})$ be a scatter matrix (functional), that is, $\boldsymbol{S}(\boldsymbol{x})$ is a symmetric positive definite $p \times p$ matrix with the affine equivariance property: $\boldsymbol{S}(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}) = \boldsymbol{A}\boldsymbol{S}(\boldsymbol{x})\boldsymbol{A}'$. In the talk we show that the independent components may be found using two separate scatter matrices.