Diophantine approximations with Fibonacci numbers

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We prove a result on Diophantine approximations with Fibonacci numbers. Let $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ be Fibonacci numbers.

Define $d_N^K = \max_{\alpha \in R} \min_{k=K...K+N-1} ||F_k\alpha||$.

Let ϕ be the Golden ratio. We prove that

1) $\lim_{N\to\infty} d_N^1 = (\phi - 1)/(\phi + 2)$;

2) $\lim_{K\to\infty} d_N^K = 1/5$, for all $N \geq 5$.

The point where d_N^K attains its maximum tends to $\alpha = 1/(\phi + 2)$ in both cases.