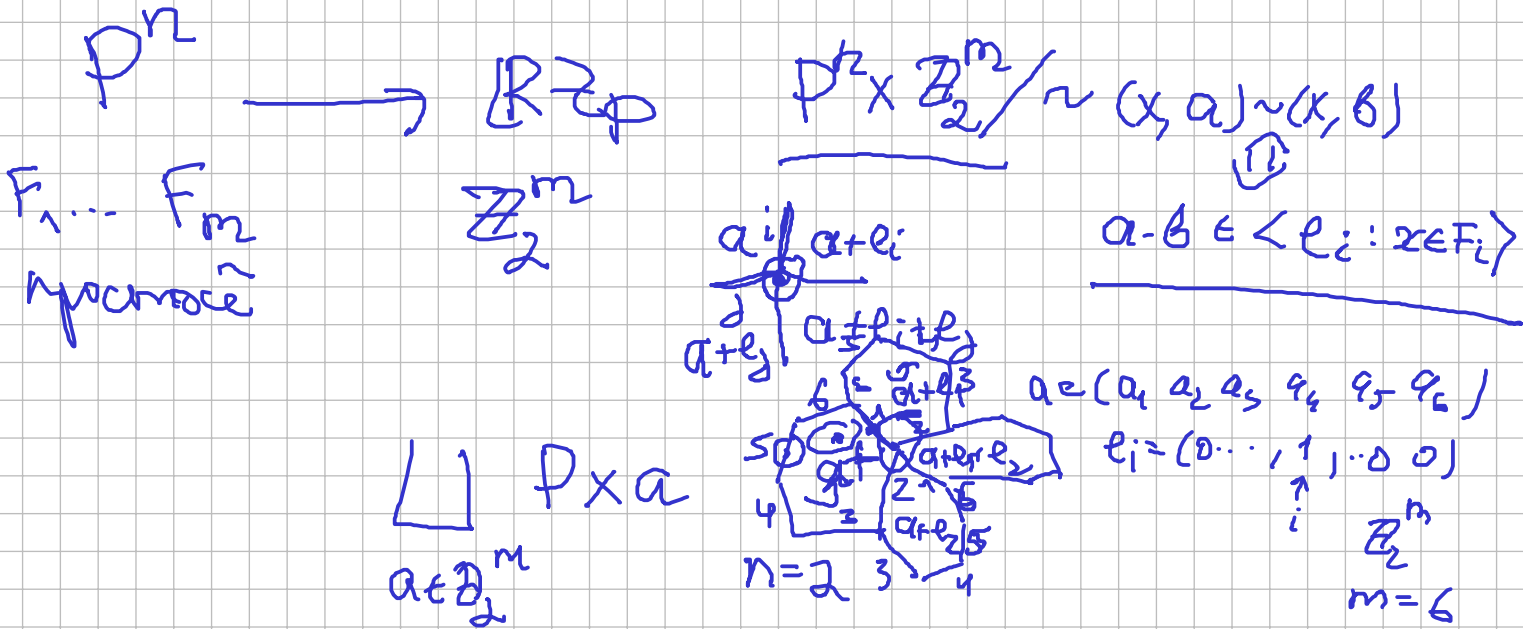


Мат-ия, опрег. несвободными
генераторами \mathbb{Z}_2^k на $\mathbb{R}\mathbb{Z}_p$



$a = a_1 \dots a_6$ $b = b_1 \dots b_6$

$\mathbb{Z}^n - b$ вершины $\mathbb{Z}^m - b$ верш

\mathbb{Z}^{m-n} вершины в $\mathbb{R}\mathbb{Z}_p$

вершины в $\mathbb{R}\mathbb{Z}_p$

вершины в $\mathbb{R}\mathbb{Z}_p$

$\mathbb{Z}^n \times \mathbb{Z}^m / \sim$ Дэтес - Деминский 9)

~ 85 А.Д. Мельник А.Ю. Весник

P^n преобразование \hat{a}

$\mathcal{Q}(P) = \langle p_1 \dots p_n \rangle$

$X \supset P$ $X / \mathcal{Q}(P) = P$

$\mathbb{R}^n \xrightarrow{\varphi} \mathbb{Z}_2^m$

$p_i \rightarrow e_i$ $\mathcal{Q}(P) / \text{Ker} = \mathbb{Z}_2^m$

$\text{Ker} = \mathcal{Q}(P)'$

$\text{Ker } \varphi$ генераторы свободной

$X / \text{Ker } \varphi = \mathbb{R}\mathbb{Z}_p$ с геометрией

$$G(P) \xrightarrow{\varphi_1} \mathbb{Z}_2^r$$

↑
неизоморфизм

$$f_i \rightarrow \lambda_i \in \mathbb{Z}_2^r$$

$$St_X = \langle f_{i_1}, \dots, f_{i_n} \rangle = \mathbb{Z}_2^n$$

$$\ker \varphi \subset G(P)$$

генераторы дообразов на X

$$St_X \cap \ker \varphi \xrightarrow{\lambda} \mathbb{Z}_2^r$$

$$\forall \text{ векторы } x = f_{i_1} \cap \dots \cap f_{i_n}$$

$$\lambda_{i_1}, \dots, \lambda_{i_n} \text{ ИДЗ}$$

$$\begin{bmatrix} \lambda_{i_1} \\ \dots \\ \lambda_{i_n} \end{bmatrix}$$

$$X / \ker \varphi_1 = \underline{N(P, \lambda)} \text{ — векторное пространство}$$

$$P \subset \mathbb{Z}^k$$

может иметь векторы не делители

$$\underline{N(P, \lambda)}$$

$$\begin{array}{ccc} G(P) & \xrightarrow{\varphi_0} & \mathbb{Z}_2^m \\ & \searrow \varphi_1 & \downarrow \varphi_1 \\ & & \mathbb{Z}_2^r \end{array}$$

$\ker \tilde{\varphi}$ генераторы на $\mathbb{R}\mathbb{Z}_P$

$$N(P, \lambda) = \mathbb{R}\mathbb{Z}_P / \ker \tilde{\varphi}_1$$

$$P \rightarrow \mathbb{R}\mathbb{Z}_P$$

$$\lambda = (\lambda_1 \dots \lambda_m)$$

$$H \subset \mathbb{Z}_2^m \xrightarrow{\lambda} \mathbb{Z}_2^r$$

$$\downarrow \varphi_1 \\ \mathbb{Z}_2^r$$

$$H(\lambda)$$

$$\mathbb{R}\mathbb{Z}_P / H(\lambda) = N(P, \lambda)$$

$$\forall \theta = f_{i_1} \cap \dots \cap f_{i_n}$$

$\lambda_{i_1}, \dots, \lambda_{i_n}$ ИДЗ \Rightarrow дообразов θ —

$$N(P, \lambda) \text{ — векторное}$$

Пусть λ — произв. элемент \mathbb{Z}_2^m тогда образ $\varphi_1(\lambda)$ — вектор

$\mathbb{R}^m \subset \mathbb{R}^m$ бex cтp ~ мeншeгo = H_0

$(x_1 + \dots + x_m = 0)$

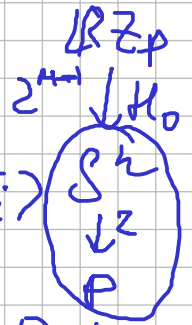
$(x_1, \dots, x_m) \in \mathbb{R}^m$

$(\mathbb{R}^2)_0 / H_0 = \mathbb{R}^2 / \mathbb{R} = \mathbb{S}^1$

$(\mathbb{R}^2)_0 / H_0 = \mathbb{R}^2 / \mathbb{R} = \mathbb{S}^1$

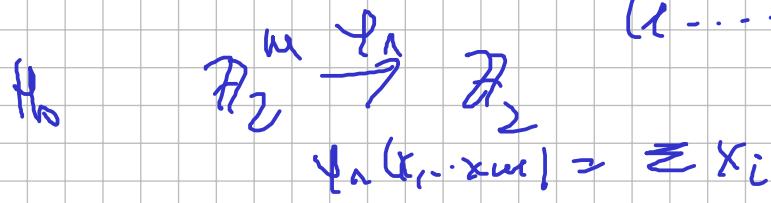
$\mathbb{R}^m / \text{ker } \varphi = \mathbb{R}^r$

$a-b \in \langle \frac{1}{c_i} : x \in F_i \rangle$



$(1, \dots, 1) = 1_0$

$\mathbb{R}^0 \times \mathbb{R}^1$

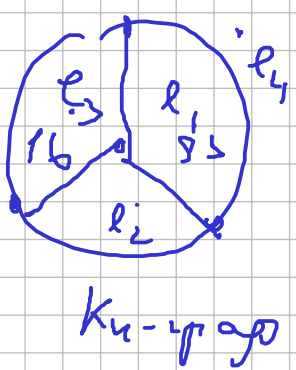
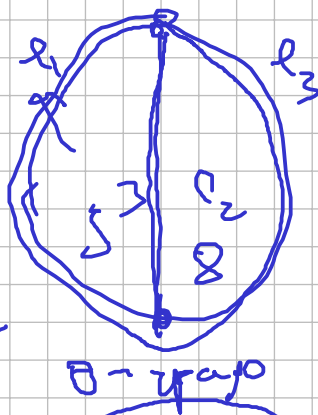
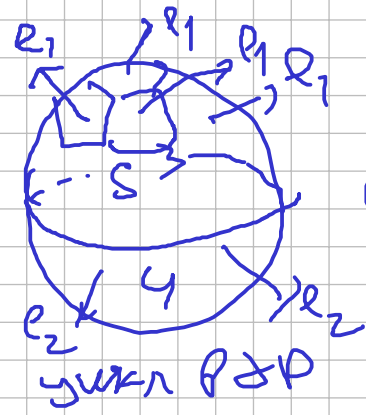
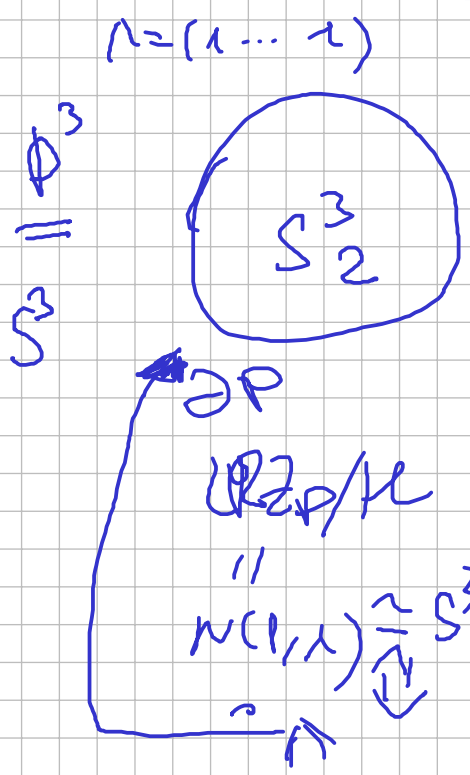


$N(P, 1_0) = \mathbb{R}^m / \mathbb{R}$

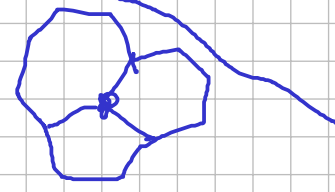
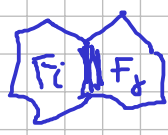
$(x, a) \sim (x, b) \iff a-b \in \langle \frac{1}{c_i} : x \in F_i \rangle$
 $(x, 0) \sim (x, 1)$

$M \subset \mathbb{R}^m \implies (\mathbb{R}^2)_0 / M \cong \mathbb{S}^1$

$H_0 \implies \sum x_i = 0$



$M/G \cong \mathbb{S}^1$

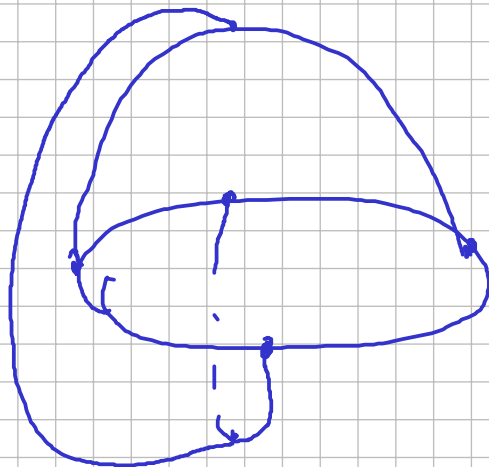
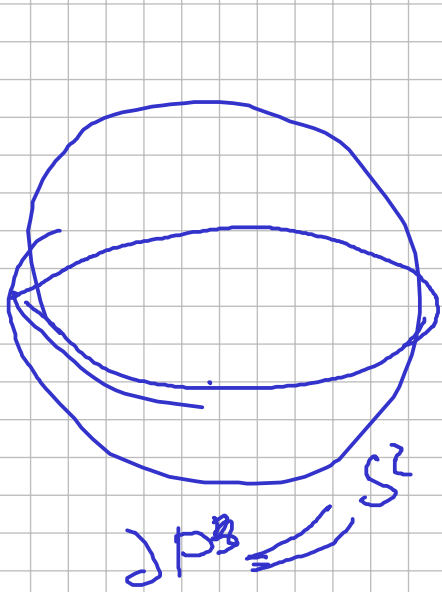


\mathbb{R}^2 Армcпoу2

$G : M$
 \cup сyмeтp.
 $H = \langle \mathbb{R} \langle x \rangle \rangle$
 $x \in M$

$\mathbb{R}^2 / H \cong \mathbb{R}^2 / \mathbb{R} \cong \mathbb{S}^1$

Максeт \rightarrow x ap бeн

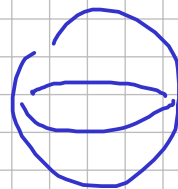


M
 \downarrow
 S^3

$P^{n-1} \cong S^{n-1}$

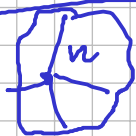
$x_1^2 + \dots + x_n^2 = 1$
 $x_1 \geq 0 \dots x_n \geq 0$

S^{n-1}
 $x_1 \geq 0$
 $x_2 \geq 0$



S^{n-1} S^k $1 \leq k \leq n$
узлы S^k
 P_i S^{k-1}

$x_1 \geq 0 \dots x_n \geq 0$



$\rightarrow k=n$

$F_{i-1} \cap F_i = F_{i-1}$
 $e_1 \dots e_{n-1}$
 F_i
 e_n
 S^{n-1}

Гиперплоскости n -мер.

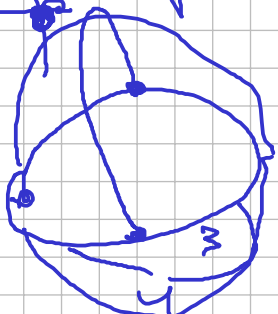
$M^n \cong$

$M^n / \tau = S^n$

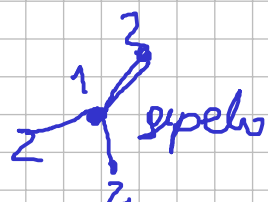
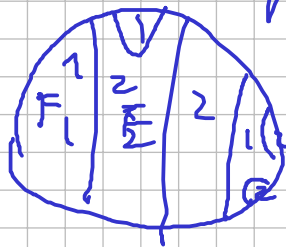
$y^2 = p(x)$

P^3

гиперплоскость S^2 содержит все вершины.



$= P^3 = S^2$



$$a_1 + b_1 = a_2 + b_2 = c$$

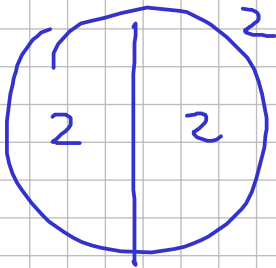
$\begin{matrix} e_1 & e_2 & e_3 & e_1+e_2+e_3 \end{matrix}$

$$\mathbb{Z}_2^3 \ni c \quad \mathbb{Z}_2^3 / \mathcal{H} = \mathbb{Z}_2^2$$

$$N(P, \mathcal{H})$$

$$\mathbb{Z}_2^3$$

$$N(P, \mathcal{H}) / \mathcal{H}$$



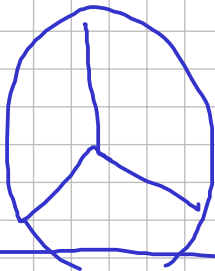
$$a_1 + b_1 = a_2 + b_2 = a_3 + b_3 = c$$

$\begin{matrix} e_1 & e_4 & e_2 & e_3 \end{matrix}$

$$\mathbb{Z}_2^4$$

$$M$$

$$N(P, \mathcal{H}) / \mathcal{H} = S^3$$



$$a_1 + b_1 = a_2 + b_2 = a_3 + b_3 = a_4 + b_4 = c$$

$\begin{matrix} e_1 & e_5 & e_2 & e_3 & e_4 \end{matrix}$

$$N(P, \mathcal{H}) / \mathcal{H} = S^3$$

$$N(P, \mathcal{H}) = 16$$

$$N(P, \mathcal{H})$$

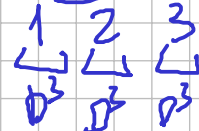
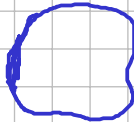
$$16$$

$$\mathbb{Z}_2^4$$

$$\begin{matrix} \mathbb{Z}_2^3 \\ \mathbb{Z}_2^2 \end{matrix}$$

$$(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2) = c$$

$$x_1^2 \geq 0, x_2^2 \geq 0, x_3^2 \geq 0$$



$$\mathbb{Z}_2^4$$

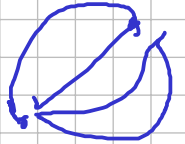
Туннель: use a_1, a_2

~~\mathbb{Z}_2^3~~ for separation area

$$\mathbb{Z}_2^4$$

$$\mathbb{Z}_2^5$$

$$\mathbb{Z}_2^6$$



$$\mathbb{Z}_2^5$$

$$\mathbb{Z}_2^6$$

$$K_5$$

\mathbb{P}



$$\mathbb{Z}_2^4$$

$$120 - \text{unit}$$



$$\frac{m-y}{a \ b} \begin{matrix} e_4 & e_5 \end{matrix}$$



$$\mathbb{P}^n$$

$$n \geq 4$$

$$\mathbb{R} \mathbb{Z}_2^m / \mathcal{H}$$

свободно

$$\mathbb{Z}_2^m / \mathcal{H} = \mathbb{Z}_2^h$$

$$N(P, \mathcal{H}) / \mathcal{H} = S^h$$

$$\mathbb{Z}_2^3$$

$$N(P, \mathcal{H})$$

$$\mathbb{Z}_2^r$$

$$\begin{matrix} \Lambda_1 & \Lambda_1 & \Lambda_1 \\ \Lambda_1 & \Lambda_2 & \Lambda_2 \\ \Lambda_1 & \Lambda_2 & \Lambda_3 \end{matrix}$$

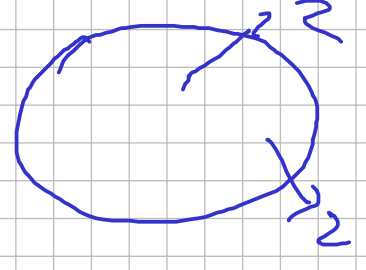
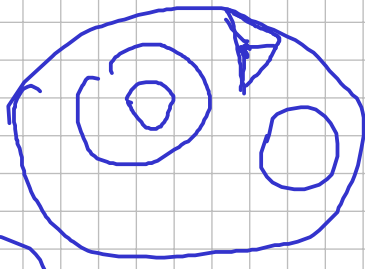
~~$$\Lambda_1$$~~

~~$$\Lambda_1$$~~

$$\Lambda_1 \ \Lambda_2 \ \Lambda_1 + \Lambda_2$$

Описание \mathbb{R}^2 универсальным в \mathbb{R}^r

$N(P, \lambda) / \alpha = S^3 \xrightarrow{\tau} \mathbb{R}^2$
 универсальность



$N(P, \lambda)$

τ универсальность $\in \mathbb{R}^r$

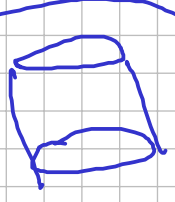


$\emptyset \quad \circ \quad \ominus \quad K_4$
 $\in \mathcal{G}^1(P, \lambda)$

$\cong \mathbb{S}^0$

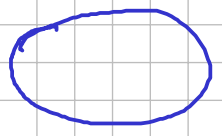
универсальность

1 2



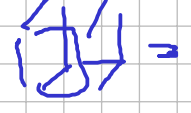
Kotzig

\mathbb{P}^3

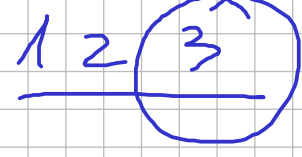


универсальность

Perfect 1-factorization



$N(P, \lambda)$

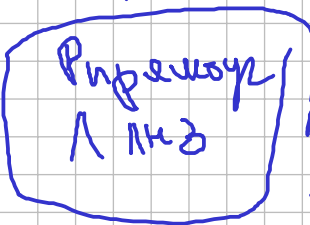


$N(P, \lambda)$

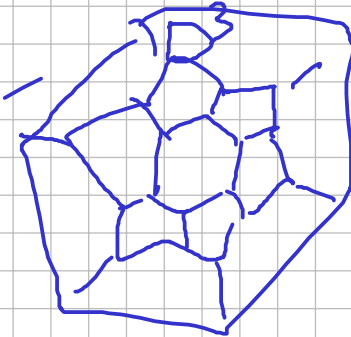
3 (2)

Браунст универсальность

(P, λ)



$N(P, \lambda)$ универсальность



$H^*(N(P, \lambda), \mathbb{Q})$

$\cong H^*(S^3, \mathbb{Q})$